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A Production Inventory Model in a Random Planning Horizon Incorporating Learning Effects on Inventory Costs Under Inflation and Time Value of Money via Genetic Algorithm

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Abstract

A production inventory model for a new product is developed incorporating inflation and time value of money. It is assumed that lifetime of the product is random in nature and follows exponential distribution with a known mean. So planning horizon of the model is random in nature. Here learning effect on production and set-up cost is incorporated. Model is formulated to maximize the expected profit from the whole plan-ning horizon and is solved using Genetic Algorithm (GA). The model is illustrated with some numerical data. Sensitivity analysis on expected profit function is also presented.

Keywords: *Learning Effect, inflation, time value of money, random planning horizon, genetic algorithm*

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INTRODUCTION

In the existing literature of inventory control problems it is implicitly assumed that life-time of the product is infinite and models are developed under finite or infinite planning horizon [1–3]. But for real life inventory problems, infinite lifetime of a product is of rare occurrence because the costs are likely to vary disproportionately and because of change in product specifications and design or its abandonment or substitution by another product due to rapid development of technology [4]. Again assumption of finite planning horizon is not appropriate if it is crisp in nature, e.g., for a seasonal product, though planning horizon is normally assumed as finite and crisp, it fluctuates in every year de-pending upon the environmental effects and it is better to estimate this horizon as fuzzy or stochastic in nature. Moon and Yun [5] developed an Economic Order Quantity (EOQ) model in random planning horizon. Recently Roy et al. [6] and Maiti et al. [7] developed an inventory model with stock dependent demand and two storage facilities over a random planning horizon. Till now, none has developed EPQ model incorporating lifetime of a product as random in nature.

Production cost of a manufacturing system depends upon the combination of different production factors. These factors are (a) raw materials, (b) technical knowledge, (c) pro-duction procedure, (d) firm size, (e) quality of product etc. Normally raw material costs are imprecise in nature. In the existing literature cost for technical knowledge, i.e., labor costs are usually assumed as constant. However, in many realistic situations, because of the firms and employees perform the same task repeatedly; they learn how to perform repeatedly. Therefore, processing cost of per unit product decreases in every cycle. Simi-larly part of ordering cost may also decrease in every cycle. This phenomenon is known as the learning effect in the literature. Although different types of learning effects have been studied in various areas [8], it has rarely been studied in the context of inventory control problems. Effect of inflation and time value of money in inventory problems is well established.

The initial attempt in this direction was made by Buzacott [9]. He dealt with an EOQ model with inflation subject to different types of pricing policies. In the subsequent year, Bierman showed that the inflation rate does not affect the optimal order quan-tity per se; rather, the difference between the inflation rate and the discount rate affects the optimum order quantity. Though a considerable number of research work has been done in this area none has consider this effect on EPQ model, especially when lifetime of the product is random.

Incorporating the above shortcomings, here an EPQ model of an item is developed in a random planning horizon, i.e., lifetime of the product is assumed as random in na-ture and it follows an exponential distribution with known mean. Unit production cost decreases in each production cycle due to learning effects of the workers on production. Similarly setup cost in each cycle is partly constant and partly decreases in each cycle due to learning effects of the employees. Model is formulated to maximize the expected profit from the whole planning horizon and is solved using Genetic Algorithm (GA). The model is illustrated with some numerical data. Sensitivity analysis on expected profit function is also presented.

GENETIC ALGORITHM

After development of Genetic Algorithm (GA) by Holland [10–12], it has been extensively used/modified to solve complex decision making problems in different field of science and technology. A GA normally starts with a set of potential solutions (called initial popula-tion) of the decision making problem under consideration. Individual solutions are called chromosome. Crossover and mutation operations happen among the potential solutions to get a new set of solutions and it continues until terminating conditions are encountered. The following functions and values are adopted in the proposed GA to solve the problem.

Parameters: The different parameters on which this GA depends are the number of generation (MAXGEN), population size (POPSIZE), probability of crossover (PCROS), probability of mutation (PMUTE).

Chromosome representation: An important issue in applying a GA is to design an appropriate chromosome representation of solutions of the problem together with genetic operators. Traditional binary vectors used to represent the chromosome are not effective in many non-linear problems. Since the proposed problem is highly non-linear, hence to overcome the difficulty, a real-number representation is used. In this representation, each chromosome V_i is a string of n number of genes G_{ij}, (j=1,2,,n) where these n number of genes, respectively denote n number of decision variables(X_i, i=1,2,.,POPSIZE).

Initial population production: For each chromosome V_i, every gene G_{ij} is randomly generated between its boundary (LB_j, UB_j) where LB_j and UB_j are the lower and upper bounds of the variables X_j, j=1,2,...,n and i=1,2,...,POPSIZE.

Evaluation: Evaluation function plays the same role in GA as that which the environ-ment plays in natural evolution. Now, evaluation function (EVAL) for the chromosome V_i is equivalent to the objective function PF(X). These are the following steps of evaluation:

1. Find EVAL(V_i) by EVAL(V_i)=f(X_1, X_2, ., X_n), where the genes G_{ij} represent the decision variable X_j, j=1,2,..,POPSIZE and f is the objective function.
2. Find total fitness of the population $F=\sum_{i=1}^{POPSIZE} \mathrm{EVAL}(V_i)$.
3. The probability p_i of selection for each chromosome V_i is determined by the formula:
$$p_i = \frac{\mathrm{EVAL}(V_i)}{F}.$$
4. Calculate the cumulative probability Y_i of selection for each chromosome V_i by the formula:
$$Y_i = \sum_{j=1}^{i} p_j.$$

Selection

The selection scheme in GA determines which solutions in the current pop-ulation are to be selected for recombination. Many selection schemes, such as Stochastic random sampling, Roulette wheel selection have been proposed for various problems. In this paper, we adopt the roulette wheel selection process.

This roulette wheel selection process is based on spinning the roulette wheel POPSIZE times, each time we select a single chromosome for the new population in the following way:

(a) Generate a random(float) number r between 0 and 1.
(b) If $r < Y_1$ then the first chromosome is V_1 otherwise select the i-th chromosome V_i ($2 \le i \le$ POPSIZE) such that Y_i-$1 \le r < Y_i$.

Crossover

Crossover operator is mainly responsible for the search of new strings. Crossover operates on two parent solutions at a time and generates offspring solutions by recombining both parent solution features. After selection chromosomes for new pop-ulation, the crossover operator is applied. Here, the arithmetic crossover operation is used.

It is defined as a linear combination of two consecutive selected chromosomes V_m and V_n and resulting offspring's V'_m and V'_n are calculated as:

$V'_m = c.V_m + (1-c).V_n$

$V'_n = c.V_n + (1-c).V_m$

where, c is a random number between 0 and 1.

Mutation

Mutation operator is used to prevent the search process from converging to lo-cal optima rapidly. It is applied to a single chromosome V_i. The selection of a chromosome for mutation is performed in the following way:

1. Set $i \leftarrow 1$
2. Generate a random number u from the range [0,1].
3. If u < PMUTE, then we select the chromosome V_i.
4. Set $i \leftarrow i+1$
5. If $i \le$ POPSIZE then go to step 2. Then the particular gene G_{ij} of the chromosome V_i selected by the above-mentioned steps is randomly selected. In this problem, the mutation is defined as G^{mut}_{ij} = random number from the range (LB_j, UB_j)

Termination

If the number of iteration is less than or equal to MAXGEN then the process is going on, otherwise it terminates.

Proposed GA procedure

```
Start
{
t ← 0
while(all constraints are not satisfied)
{
initialize Population(t)
}
evaluate Population(t)
while(not terminate-condition)
{
t ← t+1
select Population(t) from Population(t-1)
crossover and mutate Population(t)
evaluate Population(t)
}
Print Optimum Result
}
end.
```

ASSUMPTIONS AND NOTATIONS

The Mathematical model in this paper is developed on the basis of following assumptions and notations:

Assumptions:

1. Demand rate is assumed as constant.
2. The time horizon(a random variable) is finite.
3. The time horizon fully accommodates first N cycles and end during (N+1) cycles.
4. Lead time is negligible.
5. Production rate is finite.
6. Shortages are not allowed.

Notations:

1. $q(t)$= On hand inventory of a cycle at time t, $(j-1)T \le t \le jT$ (j=1,2,...,N).
2. t_1= Production time period in each cycle.
3. P = Production rate in each cycle.
4. D= Demand rate in each cycle.
5. C_1= Holding cost per unit item per unit time.
6. $C_3^j = C_3 + C_3' e^{-\beta j}$ is setup cost in j-th(j=1,2,...,N) cycle.
7. $p_0 e^{-\gamma j}$ = Production cost in j-th(j=1,2,...,N) cycle, p_0, $\gamma > 0$.
8. $m_0 p_0 e^{-\gamma j}$= Selling price in j-th(j=1,2,...,N) cycle, m_0, p_0, $\gamma > 0$.
9. N=Number of fully accommodated cycles to be made during the prescribed time horizon.
10. T = Duration of a complete cycle.

11. i = Inflation rate.
12. r= Discount rate.
13. R= r-i.
14. $P(N, T)$= Total profit after completing N fully accommodated cycles.
15. H= Total time horizon(a random variable) and h is the real time horizon.
16. HC_L= Holding cost in last cycle.
17. SR_L= Sales revenue in last cycle.
18. s_1= Reduced selling price in last cycle.
19. $E\{TP_L(T\ )\}$= Expected total profit from last cycle.
20. $E(TP)$=Expected total profit from the planning horizon.

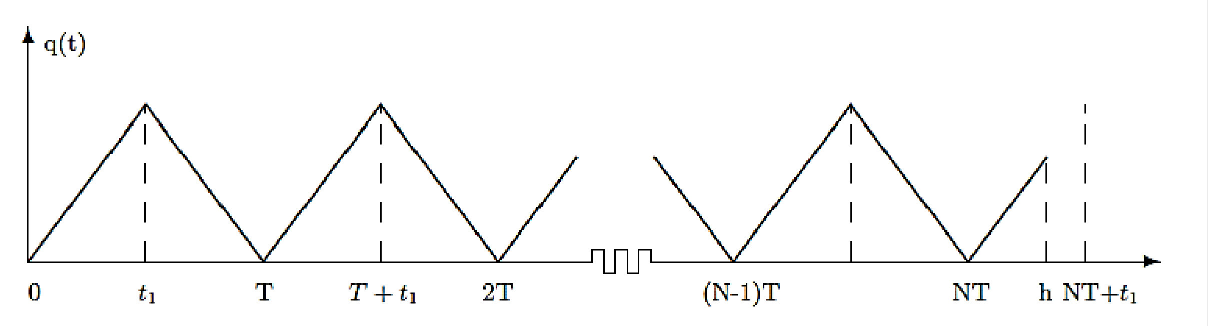

Fig. 1(a): *Inventory Level when Nt < h < Nt + t_1.*

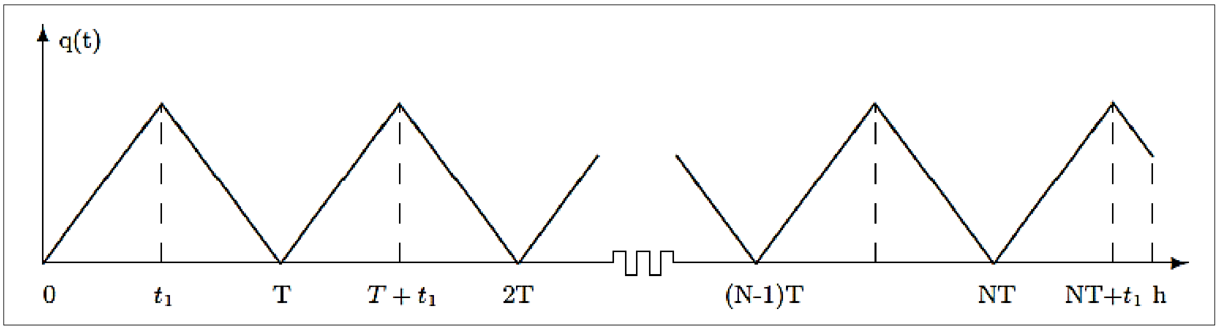

Fig. 1(b): *Inventory Level when Nt + t_1 < h < (N + 1)t.*

MATHEMATICAL FORMULATION

In the development of the model, we assume that there are N full cycles during the real time horizon h and the planning horizon ends within (N + 1)th cycle, i.e., within $t = NT$ and $t = (N+1)T$. At the beginning of every j^{th} $(j = 1, 2, ...N + 1)$ cycle production starts at t = (j − 1)T and up to t = (j − 1)T + t_1, inventory gradually increases after meeting the demand due to production (cf. Figure 1(a) and 1(b)). The inventory falls to zero level at the time $t = jT$, as the stock is depleted at the rate of D. This cycle repeats again and again. For the last cycle some amount may be left after the end of planning horizon. This amount is sold at a reduced price in a lot.

Formulation for $j^{th}(1 \le j \le N)$ Cycle: The differential equations describing the inventory level q(t) in the interval $(j-1)T \le t \le jT\ (1 \le j \le N)$ are given by,

$$\frac{dq(t)}{dt} = P - D, \quad (j-1)T \le t \le (j-1)T + t_1 \tag{1}$$

$$\frac{dq(t)}{dt} = -D, \quad (j-1)T + t_1 \le t \le jT \tag{2}$$

where, α, β, θ > 0 and $0 < t_1 < T$, subject to the conditions that, $q(t) = 0$ at $t = (j-1)T$,
and
$q(t) = 0$ at $t = jT$.

Now, DT=Pt_1=⇒ $t_1 = \frac{DT}{P}$.

The solutions of the differential Eqs. (1) and (2) are given by,

$$q(t) = \begin{cases} (P-D)t, & (j-1)T \le t \le (j-1)T + t_1 \\ D(T-t), & (j-1)T + t_1 \le t \le jT \end{cases} \tag{3}$$

Present value of holding cost of the inventory for the $j^{th}(1 \le j \le N)$ cycle is given by,

$$HC_j = \quad C_1 \int_{(j-1)T}^{(j-1)T+t_1} q(t)e^{-Rt}dt + C_1 \int_{(j-1)T+t_1}^{jT} q(t)e^{-Rt}dt$$

$$\begin{aligned}= \quad & -C_1(P-D)\left\{\frac{(j-1)T+t_1}{R}+\frac{1}{R^2}\right\}e^{-R\{(j-1)T+t_1\}} \\ & +C_1(P-D)\left\{\frac{jT}{R}+\frac{1}{R^2}\right\}e^{-RjT}+\frac{C_1.D.T}{R}\left\{e^{-R\{(j-1)T+t_1\}}-e^{-R(j-1)T}\right\} \\ & +C_1.D\left[\left\{\frac{jT}{R}+\frac{1}{R^2}\right\}e^{-RjT}-\left\{\frac{(j-1)T+t_1}{R}+\frac{1}{R^2}\right\}e^{-R\{(j-1)T+t_1\}}\right]\end{aligned} \tag{4}$$

Present value of production cost for the $j^{th}(1 \leq j \leq N)$ cycle is given by,

$$\begin{aligned}PC_j \quad & = p_0 e^{-\gamma j}.P \int_{(j-1)T}^{(j-1)T+t_1} e^{-Rt}dt \\ & = \frac{p_0 e^{-\gamma j}.P}{R}\left\{e^{-R(j-1)T-e^{-R\{(j-1)T+t_1\}}}\right\}\end{aligned} \tag{5}$$

Present value of ordering cost for the $j^{th}(1 \leq j \leq N)$ cycle is given by,

$$C_3^j = C_3 + C_3'.e^{-\beta j}.e^{-R(j-1)T},\ C_3,\ C_3',\ \beta > 0. \tag{6}$$

Present value of sales revenue for the $j^{th}(1 \leq j \leq N)$ cycle is given by,

$$\begin{aligned}SR_j = \quad & m_0.p_0.e^{-\gamma j}\int_{(j-1)T}^{jT} D.e^{-Rt}dt, m_0 > 1 \\ = \quad & \frac{m_0.p_0.e^{-\gamma j}.D}{R}\left\{e^{-R(j-1)T}-e^{-RjT}\right\}\end{aligned} \tag{7}$$

$$\text{So, } P(N,T) = \quad \sum_{j=1}^{N} SR_j - \sum_{j=1}^{N}(PC_j + HC_j + C_3^j) \tag{8}$$

$$Now,\ \sum_{j=1}^{N} e^{-R(j-1)T} = \left(\frac{1-e^{-NRT}}{1-e^{-RT}}\right) \tag{9}$$

So,

$$\begin{aligned}P(N,T) \quad & = \frac{m_0.p_0.D}{R}(e^{RT}-1).e^{-(\gamma+RT)}.\left\{\frac{1-e^{-N(\gamma+RT)}}{1-e^{-(\gamma+RT)}}\right\} \\ & -\frac{P.p_0}{R}.e^{RT}(1-e^{-R.t_1}).e^{-(\gamma+RT)}.\left\{\frac{1-e^{-N(\gamma+RT)}}{1-e^{-(\gamma+RT)}}\right\} \\ & +C_1.P\left[\frac{T}{R}\left\{\frac{(e^{-RT}-e^{-RNT})}{(1-e^{-RT})^2}-\frac{(N-1)e^{-RNT}}{(1-e^{-RT})}\right\}e^{-R.t_1}+\frac{t_1}{R}.e^{-R.t_1}\left(\frac{1-e^{-NRT}}{1-e^{-RT}}\right)\right. \\ & \left.+\frac{1}{R^2}.e^{-R.t_1}\left(\frac{1-e^{-NRT}}{1-e^{-RT}}\right)\right]-C_1(P-D)\left[\frac{T}{R}\left\{\frac{(e^{-RT}-e^{-RNT})}{(1-e^{-RT})^2}-\frac{(N-1)e^{-RNT}}{(1-e^{-RT})}\right\}\right. \\ & \left.+\frac{1}{R^2}\left(\frac{1-e^{-NRT}}{1-e^{-RT}}\right)\right]-\frac{C_1.D.T}{R}(e^{-R.t_1}-e^{-RT}).\left(\frac{1-e^{-NRT}}{1-e^{-RT}}\right) \\ & -C_1.D\left[\frac{T}{R}\left\{\frac{(e^{-RT}-e^{-R(N+1)T})}{(1-e^{-RT})^2}-\frac{Ne^{-R(N+1)T}}{(1-e^{-RT})}\right\}+\frac{e^{-R.T}}{R^2}.\left(\frac{1-e^{-NRT}}{1-e^{-RT}}\right)\right] \\ & -N.C_3-C_3'.e^{-\beta}\left(\frac{1-e^{-N(\beta+RT)}}{1-e^{-(\beta+RT)}}\right)\end{aligned} \tag{10}$$

Here, we consider the planning horizon H is a random variable, follows exponential dis-tribution with p.d.f as:

$$f(h) = \begin{cases} \lambda e^{-\lambda h}, & h \geq 0 \\ 0, & otherwise \end{cases} \tag{11}$$

Since the planning horizon H has a p.d.f f(h), the present value of expected total profit from N complete cycles is given by,

$$\begin{aligned}
E\{P(N,T)\} &= \sum_{N=0}^{\infty} \int_{NT}^{(N+1)T} P(N,T).\ f(h)\,dh \\
&= \left\{ \frac{m_0.p_0.D}{R}(e^{RT}-1) - \frac{P.p_0}{R}.e^{RT}(1-e^{-R.t_1}) \right\}.e^{-(\gamma+RT)} \times \\
&\left[\frac{1-\left(\frac{1-e^{-\lambda T}}{1-e^{-(\gamma+RT+\lambda T)}}\right)}{1-e^{-(\gamma+RT)}} \right] + C_1.P.e^{-R.t_1}\left[\frac{T}{R}\left\{ \frac{e^{-RT}-\left(\frac{1-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right)}{(1-e^{-RT})^2} \right.\right. \\
&\left. -\left(\frac{1-e^{-\lambda T}}{1-e^{-RT}}\right)\left(\frac{e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2} - \frac{1}{(1-e^{-(R+\lambda)T})}\right) \right\} \\
&\left. +\left(\frac{t_1}{R}+\frac{1}{R^2}\right)\left\{ \frac{1-\left(\frac{1-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right)}{1-e^{-RT}} \right\} \right] - C_1.(P-D)\left[\frac{T}{R}\left\{ \frac{e^{-RT}-\left(\frac{1-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right)}{(1-e^{-RT})^2} \right.\right. \\
&\left. -\left(\frac{1-e^{-\lambda T}}{1-e^{-RT}}\right)\left(\frac{e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2} - \frac{1}{(1-e^{-(R+\lambda)T})}\right) \right\} \\
&\left. +\frac{1}{R^2}\left\{ \frac{1-\left(\frac{1-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right)}{1-e^{-RT}} \right\} \right] - \frac{C_1.D.T}{R}(e^{-R.t_1}-e^{-RT})\left\{ \frac{1-\left(\frac{1-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right)}{1-e^{-RT}} \right\}
\end{aligned}$$

$$\begin{aligned}
&-C_1.D\left[\frac{T}{R}\left\{ \frac{e^{-RT}-\left(\frac{1-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right)}{(1-e^{-RT})^2} - \frac{e^{-RT}(1-e^{-\lambda T}).e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2} \right\} \right. \\
&\left. +\frac{1}{R^2}.e^{-RT}\left\{ \frac{1-\left(\frac{1-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right)}{1-e^{-RT}} \right\} \right] - (1-e^{-\lambda T})\left[C_3.\frac{e^{-\lambda T}}{(1-e^{-\lambda T})^2} \right. \\
&\left. +\frac{C_3'.e^{-\beta}}{(1-e^{-(\beta+RT)})}\left\{ \frac{1}{1-e^{-\lambda T}} - \frac{1}{1-e^{-(\beta+RT+\lambda T)}} \right\} \right]
\end{aligned} \tag{12}$$

Formulation for Last Cycle

The differential equations describing the inventory level *q(t)* in the interval $NT < t \leq (N+1)T$ are given by,

$$\frac{dq(t)}{dt} = P - D, \quad NT \leq t \leq NT + t_1 \tag{13}$$

$$\frac{dq(t)}{dt} = -D, \quad NT + t_1 \leq t \leq (N+1)T \tag{14}$$

Subject to the conditions that,

q(NT) = 0 & q(N + 1)T = 0

The solutions of the differential Eqs. (13) and (14) are given by,

$$q(t)=\begin{cases}(P-D)t, & NT\le t\le NT+t_1\\ D(T-t), & NT+t_1\le t\le (N+1)T\end{cases} \tag{15}$$

In last cycle, we consider two cases depending upon the cycle length value Let h be the real corresponding to the random variable H.

Case-I (NT < h ≤ NT + t_1): Present value of holding cost of the inventory for the last cycle is given by,

$$\begin{aligned}HC_{L1} &= C_1\int_{NT}^{h} q(t)e^{-Rt}dt\\ &= C_1.(P-D)\left[\left\{\frac{NT}{R}+\frac{1}{R^2}\right\}e^{-RNT}-\left\{\frac{h}{R}+\frac{1}{R^2}\right\}e^{-Rh}\right]\end{aligned} \tag{16}$$

Present value of production cost is given by,

$$\begin{aligned}PC_{L1} &= p_0.e^{-\gamma(N+1)}.P\int_{NT}^{h} e^{-Rt}dt\\ &= \frac{p_0.e^{-\gamma(N+1)}.P}{R}\left[e^{-RNT}-e^{-Rh}\right]\end{aligned} \tag{17}$$

Present value of ordering cost= $C_3+C'_3.e^{-\beta(N+1).}e^{-NRT}$

Present value of sales revenue is given by,

$$\begin{aligned}SR_{L1} &= m_0.p_0.e^{-\gamma(N+1)}.D\int_{NT}^{h} e^{-Rt}dt\\ &= \frac{m_0.p_0.e^{-\gamma(N+1)}.D}{R}\left[e^{-RNT}-e^{-Rh}\right]\end{aligned} \tag{18}$$

Case-II (NT + t_1 < h ≤ (N + 1)T): Present value of holding cost of the inventory for the last cycle is given by,

$$\begin{aligned}HC_{L2} &= C_1\int_{NT}^{NT+t_1} q(t)e^{-Rt}dt + C_1\int_{NT+t_1}^{h} q(t)e^{-Rt}dt\\ &= C_1.(P-D)\left[-\left\{\frac{NT+t_1}{R}+\frac{1}{R^2}\right\}e^{-R(NT+t_1)}+\left\{\frac{NT}{R}+\frac{1}{R^2}\right\}e^{-RNT}\right]\\ &-\frac{C_1.D.T}{R}\left[e^{-Rh}-e^{-R(NT+t_1)}\right]\end{aligned} \tag{19}$$

Present value of production cost is given by,

$$\begin{aligned}PC_{L2} &= p_0.e^{-\gamma(N+1)}.P\int_{NT}^{NT+t_1} e^{-Rt}dt\\ &= \frac{p_0.e^{-\gamma(N+1)}.P}{R}\left[e^{-RNT}-e^{-R(NT+t_1)}\right]\end{aligned} \tag{20}$$

Present value of ordering cost= $C_3+C'_3.e^{-\beta(N+1).}e^{-NRT}$ Present value of sales revenue is given by,

$$\begin{aligned}SR_{L2} &= m_0.p_0.e^{-\gamma(N+1)}.D\int_{NT}^{NT+t_1} e^{-Rt}dt + m_0.p_0.e^{-\gamma(N+1)}.D\int_{NT+t_1}^{h} e^{-Rt}dt\\ &= \frac{m_0.p_0.e^{-\gamma(N+1)}.D}{R}\left[e^{-RNT}-e^{-Rh}\right]\end{aligned} \tag{21}$$

So, expected holding cost for the last cycle is given by,

$$\sum_{N=0}^{\infty}\int_{NT}^{(N+1)T} HC_L\,.f(h)dh$$
$$=\sum_{N=0}^{\infty}\int_{NT}^{NT+t_1} HC_{L1}\,.f(h)dh+\sum_{N=0}^{\infty}\int_{NT+t_1}^{(N+1)T} HC_{L2}\,.f(h)dh \tag{22}$$

Expected production cost for the last cycle is given by,

$$\sum_{N=0}^{\infty}\int_{NT}^{(N+1)T} PC_L\,.f(h)dh$$
$$=\sum_{N=0}^{\infty}\int_{NT}^{NT+t_1} PC_{L1}\,.f(h)dh+\sum_{N=0}^{\infty}\int_{NT+t_1}^{(N+1)T} PC_{L2}\,.f(h)dh \tag{23}$$

Expected sales revenue for the last cycle is given by,

$$\sum_{N=0}^{\infty}\int_{NT}^{(N+1)T} SR_L\,.f(h)dh$$
$$=\sum_{N=0}^{\infty}\int_{NT}^{NT+t_1} SR_{L1}\,.f(h)dh+\sum_{N=0}^{\infty}\int_{NT+t_1}^{(N+1)T} SR_{L2}\,.f(h)dh \tag{24}$$

Expected ordering cost for the last cycle is given by,

$$\sum_{N=0}^{\infty}\int_{NT}^{(N+1)T}\left\{C_3+C_3'.e^{-\beta(N+1)}.e^{-NRT}\right\}f(h)dh \tag{25}$$

Expected reduced selling price from the last cycle is given by,

$$s_1\sum_{N=0}^{\infty}\int_{NT}^{(N+1)T} e^{-Rh}q(h)\,.f(h)dh$$
$$=s_1\sum_{N=0}^{\infty}\int_{NT}^{NT+t_1} e^{-Rh}q(h)\,.f(h)dh+s_1\sum_{N=0}^{\infty}\int_{NT+t_1}^{(N+1)T} e^{-Rh}q(h)\,.f(h)dh \tag{26}$$

So, expected total profit from last cycle is given by,

$$\begin{aligned}
E\{TP_L(T)\} \quad &=\frac{m_0.p_0.D}{R}\left\{\frac{(1-e^{-\lambda T}).e^{-\gamma}}{1-e^{-(\gamma+RT+\lambda T)}}\right\}+\frac{m_0.p_0.D}{R}\frac{\lambda}{R+\lambda}\left\{\frac{(e^{-(R+\lambda)T}-1).e^{-\gamma}}{1-e^{-(\gamma+RT+\lambda T)}}\right\}\\
&+m_0.p_0.(P-D).\lambda.e^{-\gamma}\left[\frac{T}{(R+\lambda)}.\left\{\frac{e^{-(\gamma+RT+\lambda T)}}{(1-e^{-(\gamma+RT+\lambda T)})^2}\right\}\right]\\
&+\frac{1}{(R+\lambda)^2}.\frac{1}{(1-e^{-(\gamma+RT+\lambda T)})}-\frac{T}{(R+\lambda)}e^{-(R+\lambda)t_1}.\left\{\frac{e^{-(\gamma+RT+\lambda T)}}{(1-e^{-(\gamma+RT+\lambda T)})^2}\right\}\\
&-\left\{\frac{t_1}{R+\lambda}+\frac{1}{(R+\lambda)^2}\right\}.e^{-(R+\lambda)t_1}.\frac{1}{(1-e^{-(\gamma+RT+\lambda T)})}\\
&+m_1.p_0.e^{-\gamma}.\frac{D.T.\lambda}{(R+\lambda)}\left[\frac{e^{-(R+\lambda)t_1}}{(1-e^{-(\gamma+RT+\lambda T)})}-\frac{e^{-(R+\lambda)T}}{(1-e^{-(\gamma+RT+\lambda T)})}\right]\\
&+m_1.p_0.D.\lambda.e^{-\gamma}\left[\frac{T.e^{-(\gamma+RT+\lambda T)}}{(R+\lambda)}.\left\{\frac{e^{-(\gamma+RT+\lambda T)}}{(1-e^{-(\gamma+RT+\lambda T)})^2}+\frac{1}{(1-e^{-(\gamma+RT+\lambda T)})}\right\}\right.\\
&+\frac{e^{-(R+\lambda)T}}{(R+\lambda)^2}.\frac{1}{(1-e^{-(\gamma+RT+\lambda T)})}-e^{-(R+\lambda)t_1}.\left\{\frac{T}{(R+\lambda)}.\frac{e^{-(\gamma+RT+\lambda T)}}{(1-e^{-(\gamma+RT+\lambda T)})^2}\right.\\
&\left.\left.+\left(\frac{t_1}{R+\lambda}+\frac{1}{(R+\lambda)^2}\right)\frac{1}{(1-e^{-(\gamma+RT+\lambda T)})}\right\}\right]
\end{aligned}$$

$$
-\frac{p_0.P}{R}\left\{\frac{(1-e^{-\lambda t_1}).e^{-\gamma}}{1-e^{-(\gamma+RT+\lambda T)}}\right\}-\frac{p_0.P}{R}\frac{\lambda}{R+\lambda}\left\{\frac{(e^{-(R+\lambda)t_1}-1).e^{-\gamma}}{1-e^{-(\gamma+RT+\lambda T)}}\right\}
$$
$$
-C_1.(P-D)\left[\frac{\lambda}{R}\left\{\frac{T.e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2}.\frac{e^{-(R+\lambda)t_1}}{R+\lambda}+\frac{t_1.e^{-(R+\lambda)t_1}}{(R+\lambda).(1-e^{-(R+\lambda)T})}\right.\right.
$$
$$
\left.+\frac{e^{-(R+\lambda)t_1}}{(R+\lambda)^2.(1-e^{-(R+\lambda)T})}\right\}+\frac{e^{-(R+\lambda)t_1}}{R^2.(1-e^{-(R+\lambda)T})}-\frac{\lambda}{R}\left\{\frac{T.e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2}.\frac{1}{R+\lambda}\right.
$$
$$
\left.\left.+\frac{1}{(R+\lambda)^2.(1-e^{-(R+\lambda)T})}\right\}-\frac{1}{R^2.(1-e^{-(R+\lambda)T})}\right]
$$
$$
+C_1.(P-D)\left[\frac{T}{R}\left\{(e^{-\lambda t_1}-1)\frac{e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2}+\frac{(e^{-\lambda t_1}-1)}{R^2.(1-e^{-(R+\lambda)T})}\right]\right.
$$
$$
+C_1.(P-D)\left[\left[\left\{\frac{T}{R}.e^{-\lambda T}.\frac{e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2}-\frac{T}{R}.e^{-\lambda t_1}.\frac{e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2}\right.\right.\right.
$$
$$
\left.+\frac{(e^{-\lambda T}-e^{-\lambda t_1})}{R^2.(1-e^{-(R+\lambda)T})}\right\}-\left\{\frac{T}{R}.e^{-Rt_1-\lambda T}.\frac{e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2}\right.
$$
$$
\left.-\frac{T}{R}.e^{-(R+\lambda)t_1}.\frac{e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2}+\frac{t_1}{R}\frac{(e^{-Rt_1-\lambda T}-e^{-(R+\lambda)t_1})}{(1-e^{-(R+\lambda)T})}\right\}
$$

$$
\left.\left.-\frac{1}{R^2}\left\{\frac{e^{-\lambda T-Rt_1}}{(1-e^{-(R+\lambda)T})}-\frac{e^{-(\lambda+R)t_1}}{(1-e^{-(R+\lambda)T})}\right\}\right]\right]
$$
$$
-\frac{\lambda C_1.D.T}{R}\left[\left[\left\{\frac{e^{-(R+\lambda)T}-e^{-(R+\lambda)t_1}}{(R+\lambda)(1-e^{-(R+\lambda)T})}\right\}-\left\{\frac{e^{-Rt_1-\lambda T}-e^{-(R+\lambda)t_1}}{\lambda(1-e^{-(R+\lambda)T})}\right\}\right]\right]
$$
$$
-C_3-C_3'.e^{-\beta}.\frac{(1-e^{-\lambda T})}{(1-e^{-(\beta+\lambda T+RT)})} \tag{27}
$$

Total Profit from the System

Now, total expected profit from the complete time horizon is given by,

$$
\begin{aligned}
E(TP(T)) &= E(P(N,T))+E\{TP_L(T)\}\\
&=\left\{\frac{m_0.p_0.D}{R}(e^{RT}-1)-\frac{P.p_0}{R}.e^{RT}(1-e^{-R.t_1})\right\}.e^{-(\gamma+RT)}\times\\
&\left[\frac{1-\left(\frac{1-e^{-\lambda T}}{1-e^{-(\gamma+RT+\lambda T)}}\right)}{1-e^{-(\gamma+RT)}}\right]+C_1.P.e^{-R.t_1}\left[\frac{T}{R}\left\{\frac{e^{-RT}-\left(\frac{1-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right)}{(1-e^{-RT})^2}\right.\right.\\
&\left.-\left(\frac{1-e^{-\lambda T}}{1-e^{-RT}}\right)\left(\frac{e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2}-\frac{1}{(1-e^{-(R+\lambda)T})}\right)\right\}\\
&\left.+\left(\frac{t_1}{R}+\frac{1}{R^2}\right)\left\{\frac{1-\left(\frac{1-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right)}{1-e^{-RT}}\right\}\right]-C_1.(P-D)\left[\frac{T}{R}\left\{\frac{e^{-RT}-\left(\frac{1-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right)}{(1-e^{-RT})^2}\right.\right.\\
&\left.-\left(\frac{1-e^{-\lambda T}}{1-e^{-RT}}\right)\left(\frac{e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2}-\frac{1}{(1-e^{-(R+\lambda)T})}\right)\right\}\\
&\left.+\frac{1}{R^2}\left\{\frac{1-\left(\frac{1-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right)}{1-e^{-RT}}\right\}\right]-\frac{C_1.D.T}{R}(e^{-R.t_1}-e^{-RT})\left\{\frac{1-\left(\frac{1-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right)}{1-e^{-RT}}\right\}
\end{aligned}
$$

$$-C_1.D\left[\frac{T}{R}\left\{\frac{e^{-RT}-\left(\frac{1-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right)}{(1-e^{-RT})^2}-\frac{e^{-RT}(1-e^{-\lambda T}).e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2}\right\}\right.$$

$$\left.+\frac{1}{R^2}.e^{-RT}\left\{\frac{1-\left(\frac{1-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right)}{1-e^{-RT}}\right\}\right]-(1-e^{-\lambda T})\left[C_3.\frac{e^{-\lambda T}}{(1-e^{-\lambda T})^2}\right.$$

$$\left.+\frac{C_3'.e^{-\beta}}{(1-e^{-(\beta+RT)})}\left\{\frac{1}{1-e^{-\lambda T}}-\frac{1}{1-e^{-(\beta+RT+\lambda T)}}\right\}\right]$$

$$+\frac{m_0.p_0.D}{R}\left\{\frac{(1-e^{-\lambda T}).e^{-\gamma}}{1-e^{-(\gamma+RT+\lambda T)}}\right\}+\frac{m_0.p_0.D}{R}\frac{\lambda}{R+\lambda}\left\{\frac{(e^{-(R+\lambda)T}-1).e^{-\gamma}}{1-e^{-(\gamma+RT+\lambda T)}}\right\}$$

$$+m_0.p_0.(P-D).\lambda.e^{-\gamma}\left[\frac{T}{(R+\lambda)}.\left\{\frac{e^{-(\gamma+RT+\lambda T)}}{(1-e^{-(\gamma+RT+\lambda T)})^2}\right\}\right]$$

$$+\frac{1}{(R+\lambda)^2}.\frac{1}{(1-e^{-(\gamma+RT+\lambda T)})}-\frac{T}{(R+\lambda)}e^{-(R+\lambda)t_1}.\left\{\frac{e^{-(\gamma+RT+\lambda T)}}{(1-e^{-(\gamma+RT+\lambda T)})^2}\right\}$$

$$-\left\{\frac{t_1}{R+\lambda}+\frac{1}{(R+\lambda)^2}\right\}.e^{-(R+\lambda)t_1}.\frac{1}{(1-e^{-(\gamma+RT+\lambda T)})}$$

$$+m_1.p_0.e^{-\gamma}.\frac{D.T.\lambda}{(R+\lambda)}\left[\frac{e^{-(R+\lambda)t_1}}{(1-e^{-(\gamma+RT+\lambda T)})}-\frac{e^{-(R+\lambda)T}}{(1-e^{-(\gamma+RT+\lambda T)})}\right]$$

$$+m_1.p_0.D.\lambda.e^{-\gamma}\left[\frac{T.e^{-(\gamma+RT+\lambda T)}}{(R+\lambda)}.\left\{\frac{e^{-(\gamma+RT+\lambda T)}}{(1-e^{-(\gamma+RT+\lambda T)})^2}+\frac{1}{(1-e^{-(\gamma+RT+\lambda T)})}\right\}\right.$$

$$+\frac{e^{-(R+\lambda)T}}{(R+\lambda)^2}.\frac{1}{(1-e^{-(\gamma+RT+\lambda T)})}-e^{-(R+\lambda)t_1}.\left\{\frac{T}{(R+\lambda)}.\frac{e^{-(\gamma+RT+\lambda T)}}{(1-e^{-(\gamma+RT+\lambda T)})^2}\right.$$

$$\left.\left.+\left(\frac{t_1}{R+\lambda}+\frac{1}{(R+\lambda)^2}\right)\frac{1}{(1-e^{-(\gamma+RT+\lambda T)})}\right\}\right]$$

$$-\frac{p_0.P}{R}\left\{\frac{(1-e^{-\lambda t_1}).e^{-\gamma}}{1-e^{-(\gamma+RT+\lambda T)}}\right\}-\frac{p_0.P}{R}\frac{\lambda}{R+\lambda}\left\{\frac{(e^{-(R+\lambda)t_1}-1).e^{-\gamma}}{1-e^{-(\gamma+RT+\lambda T)}}\right\}$$

$$-C_1.(P-D)\left[\frac{\lambda}{R}\left\{\frac{T.e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2}.\frac{e^{-(R+\lambda)t_1}}{R+\lambda}+\frac{t_1.e^{-(R+\lambda)t_1}}{(R+\lambda).(1-e^{-(R+\lambda)T})}\right.\right.$$

$$\left.+\frac{e^{-(R+\lambda)t_1}}{(R+\lambda)^2.(1-e^{-(R+\lambda)T})}\right\}+\frac{e^{-(R+\lambda)t_1}}{R^2.(1-e^{-(R+\lambda)T})}-\frac{\lambda}{R}\left\{\frac{T.e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2}.\frac{1}{R+\lambda}\right.$$

$$\left.\left.+\frac{1}{(R+\lambda)^2.(1-e^{-(R+\lambda)T})}\right\}-\frac{1}{R^2.(1-e^{-(R+\lambda)T})}\right]$$

$$+C_1.(P-D)\left[\frac{T}{R}\left\{(e^{-\lambda t_1}-1)\frac{e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2}+\frac{(e^{-\lambda t_1}-1)}{R^2.(1-e^{-(R+\lambda)T})}\right]\right.$$

$$+C_1.(P-D)\left[\left\{\frac{T}{R}.e^{-\lambda T}.\frac{e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2}-\frac{T}{R}.e^{-\lambda t_1}.\frac{e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2}\right.\right.$$

$$\left.+\frac{(e^{-\lambda T}-e^{-\lambda t_1})}{R^2.(1-e^{-(R+\lambda)T})}\right\}-\left\{\frac{T}{R}.e^{-Rt_1-\lambda T}.\frac{e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2}\right.$$

$$\left.-\frac{T}{R}.e^{-(R+\lambda)t_1}.\frac{e^{-(R+\lambda)T}}{(1-e^{-(R+\lambda)T})^2}+\frac{t_1}{R}\frac{(e^{-Rt_1-\lambda T}-e^{-(R+\lambda)t_1})}{(1-e^{-(R+\lambda)T})}\right\}$$

$$\left.\left.-\frac{1}{R^2}\left\{\frac{e^{-\lambda T-Rt_1}}{(1-e^{-(R+\lambda)T})}-\frac{e^{-(\lambda+R)t_1}}{(1-e^{-(R+\lambda)T})}\right\}\right]\right]$$

$$-\frac{\lambda C_1.D.T}{R}\left[\left\{\frac{e^{-(R+\lambda)T}-e^{-(R+\lambda)t_1}}{(R+\lambda)(1-e^{-(R+\lambda)T})}\right\}-\left\{\frac{e^{-Rt_1-\lambda T}-e^{-(R+\lambda)t_1}}{\lambda(1-e^{-(R+\lambda)T})}\right\}\right]$$

$$-C_3-C_3'.e^{-\beta}.\frac{(1-e^{-\lambda T})}{(1-e^{-(\beta+\lambda T+RT)})} \tag{28}$$

Problem Formulation

So, the above problem can be formulated as:

Maximize $E(TP(T))$ (29)

subject to, P > D

NUMERICAL ILLUSTRATION

The following numerical data are used to illustrate the model.

C_3 = \$50, C_3' = \$100D = 20, P = 25, C_1 = \$1.0, γ = 0.05, β = 0.05, λ = 0.01, m_0 = 1.8, m_1 = 0.8 r = 0.1, i = 0.05, i.e., R = 0.05 in appropriate units.

The optimal values of T along with maximum expected total profit have been calcu-lated for different values of P and D and results are displayed in Table 1.

Table 1: *Results for above Inventory Model.*

P	D	T	E(TP)
25	19	22.51	195.74
	20	24.11	468.32
	21	25.88	765.12
30	24	22.86	660.49
	25	24.25	949.55
	26	25.72	1258.43

Sensitivity Analysis

Results are obtained for above parametric values and different values of β and presented in Table 2. It is observed that profit increases with β. It hap-pens due to the fact that increase of β decreases the setup cost in different cycles which in turn decreases profit.

Table 2: *Results due to Different β.*

β	T	E(TP)
0.03	24.81	443.76
0.04	24.45	456.33
0.05	24.11	468.32
0.06	23.82	479.76
0.07	23.55	490.65

Results are also obtained for above parametric values and different values of ’resultant effect of inflation and discount rate’, R, and presented in Table 3. It is observed that profit decreases with R, which agrees with reality.

Table 3: *Results due to Different R.*

R	T	E(TP)
0.051	23.83	463.92
0.052	23.58	458.78
0.053	23.36	453.01
0.054	23.15	446.69
0.055	22.97	439.88
0.056	22.82	432.65

CONCLUSION

In this paper, for the first time an EPQ model has been considered under inflation and time discounting over a stochastic time horizon. Also for the first time learning effect on production and setup cost are incorporated in an EPQ model. The methodology presented here is quite general and can be applied to the inventory problems with dynamic demand, allowing shortages, etc.

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